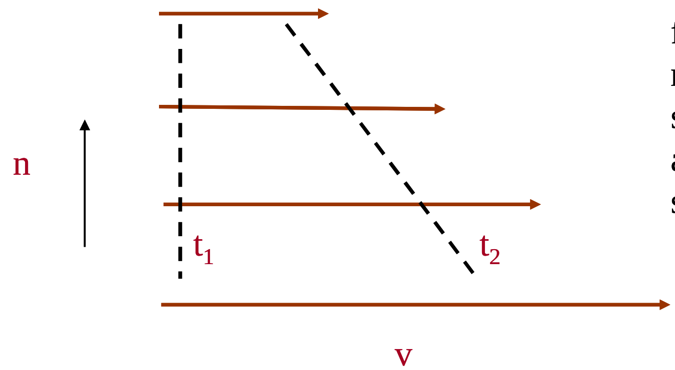
Vorticity

We have previously discussed the angular velocity as a measure of rotation of a body. This is a suitable quantity for a body that retains its shape but a fluid can distort and we must consider two components to rotation: shear and curvature. We introduce the term vorticity as a measure of rotation within a fluid.

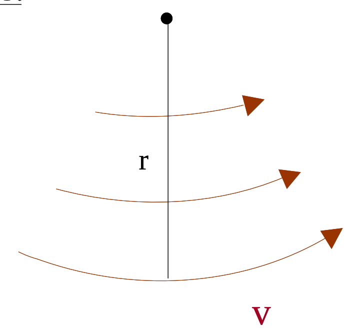
shear:



fluid elements moving in a straight line but at different speeds.

Consider a marker in the fluid such as the dashed line crossing the streamlines. Initially, the marker might be normal to the streamlines, as at time t_1, but later will be rotated to the position at t_2. The marker line has undergone an anticlockwise rotation (cyclonic in the N-hemisphere). We evaluate the contribution to the vorticity as $-\frac{dv}{dn}$

curvature:



If, at radial distance r, the velocity is V, the curvative component of vorticity is evaluated as V/r in the same way we define angular velocity.

Convention: both components are assessed the same way using the convention that cyclonic (anticlockwise in the N-hemisphere) rotation is positive, and anticyclonic rotation is negative.

The vorticity of a volume of atmosphere, when evaluated in this way from the velocity of the air relative to the earth's surface is called relative vorticity ζ (Greek zeta)

Relative vorticity $\zeta = \frac{v}{r} - \frac{dv}{dn}$

If the atmosphere rotated like a solid body, the two components would be equal and thus the vorticity is equivalent to twice the angular velocity of a solid object.

To apply basic laws of motion (such as the conservation of angular momentum), we need to consider rotation in an absolute frame of reference, and must add in the rotation due to that of the earth itself. Twice the angular velocity of the earth about a local vertical is, of course, equal to the Coriolis parameter $f = 2\Omega \sin\phi$, so we define absolute vorticity as

$$\zeta_a = \zeta + f$$

On certain upper air charts (see, for example, the NGM 500 mb forecast maps), absolute vorticity is plotted in units of $10^{-5}\ s^{-1}$. Thus, in mid latitudes, where $f \approx 10 \times 10^{-5}\ s^{-1}$, a region of zero relative vorticity would have an absolute vorticity of 10 units. Centers of maximum vorticity are useful in identifying trough locations, which are sometimes difficult to identify, otherwise, especially since the shear component is difficult to assess.

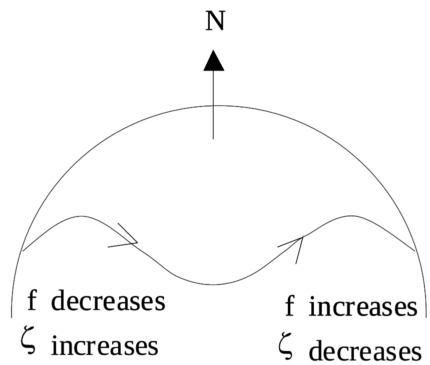
As we'll see later the relationship between the flow field and the vorticity pattern yields an important forecasting tool.

Constant absolute vorticity trajectories

It has been shown that, to a reasonable approximation, the atmosphere moves in such a manner as to conserve its absolute vorticity. That is,

$$\zeta_a = \zeta + f = \text{constatnt}$$

Atmospheric trajectories with constant absolute vorticity execute sinusoidal-like paths around the hemisphere. Remember that the Coriolis parameter f (twice the angular velocity of the earth about a local vertical) increases towards the pole ($f = 2\Omega \sin\phi$). If f increases as a mass of air moves northward, then ζ must decrease, and vice versa. The result is a wavy trajectory around the hemisphere as shown below:



To understand this, remember that positive ζ is cyclonic (anticlockwise) rotation, while negative ζ is anticyclonic (clockwise) motion, and ζ cycles between positive and negative values as the air executes a sine wave.

Rossby waves

These large wave-like perturbations observed in the mid-latitude westerly flow are called Rossby waves after the Swedish meteorologist (who founded the first meteorology department in the US at MIT in 1928). He was the first to recognize the importance of such disturbances to the global circulation pattern.

Analysis of Rossby waves results in the following expression for the speed at which the waves propagate towards the east:

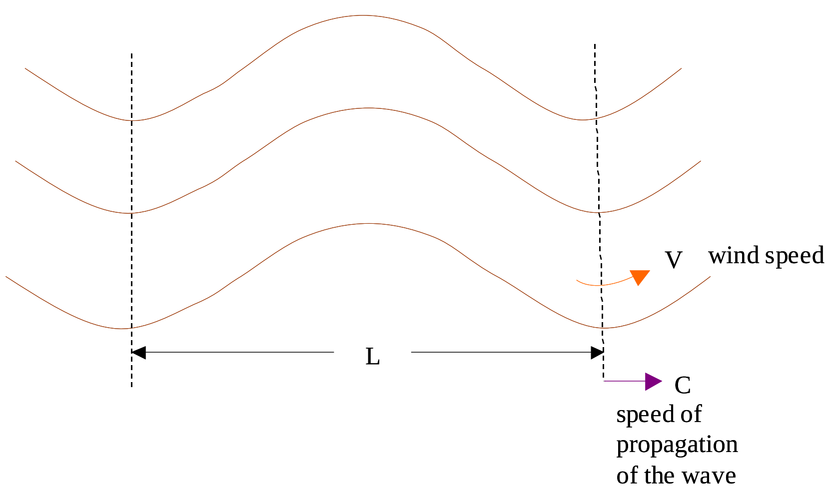
Rossby wave speed $c = v - \frac{\beta L^2}{4\pi^2}$

where V is the wind speed at mid-tropospheric levels (specifically at the level of non-divergence LND),
L is the wavelength of the Rossby wave

$$\beta = \frac{2\Omega}{a}\cos\phi$$

where

$$\Omega = 7.292\times10^{-5} \ \text{rad/s}$$
$$a = \text{radisus of earth}$$
$$= 6378 \ \text{km}$$
$$\phi = \text{latitude angle}$$



Since the last term in the equation $\left(\frac{\beta L^2}{4\pi^2}\right)$ is always positive, the wave speed C must be smaller than the wind speed. Thus, the air passes through the troughs and ridges at a speed greater than the pattern itself propagates.

Notice that the shorter the wavelength L, the larger the wave speed C. The longer the wavelength, the smaller the wave speed. Long waves seen on upper air maps thus move slowly while shorter waves ripple through them. At any one time, the global circulation pattern is a summation of waves of different length in different stages of being in or out of phase.

From the equation, one can see that a sufficiently large wave could have a negative speed of propagation (C < 0 if $\frac{\beta L^2}{4\pi^2} > V$) and the wave will retrograde (move towards the west).

Example calculations of wave speed C:

Suppose V = 30 m/s, $\phi = 45^{o}\,N$

$$\beta = \frac{2\Omega\cos\phi}{a} = \frac{(2)(7.292\times10^{-5})\cos 45}{(6.38\times10^{6})} = 1.62\times10^{-11}$$

$$\frac{\beta}{4\pi^2} = 0.405\times10^{-12}$$

for a wavelength L = 4000 km

$$\begin{aligned} C &= 30 - (0.405 \times 10^{-12})(4 \times 10^{6})^{2} \\ &= 30 - 6.4 = 23.6 \text{ m/s} \end{aligned}$$

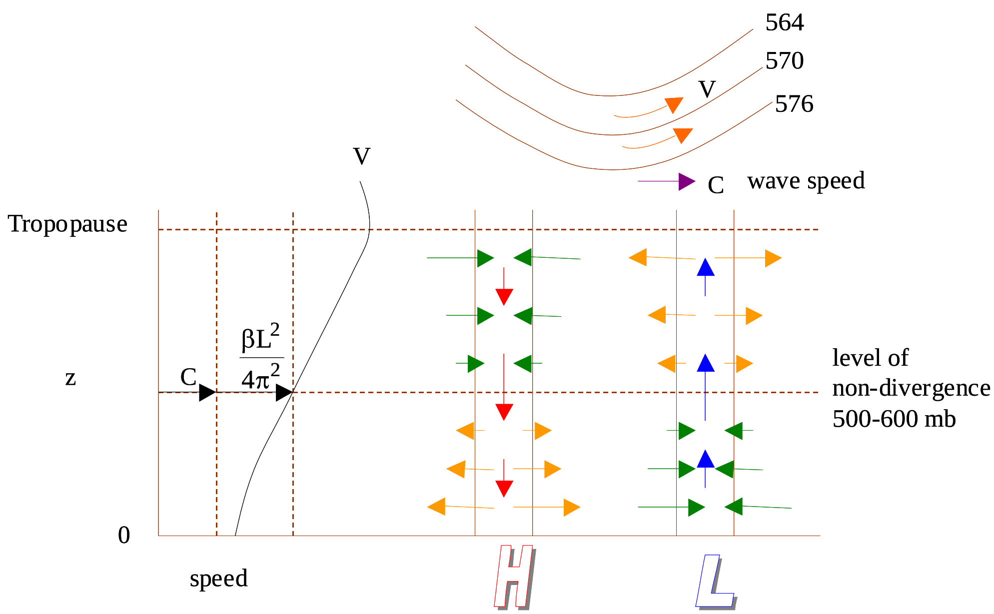
for a wavelength L = 8000 km

$$\begin{aligned} C &= 30 - (0.405 \times 10^{-12})(8 \times 10^{6})^{2} \\ &= 30 - 25.9 = 4.1 \text{ m/s} \end{aligned}$$

Thus, the longer wave propagates towards the east at a much lower rate than the short wave.

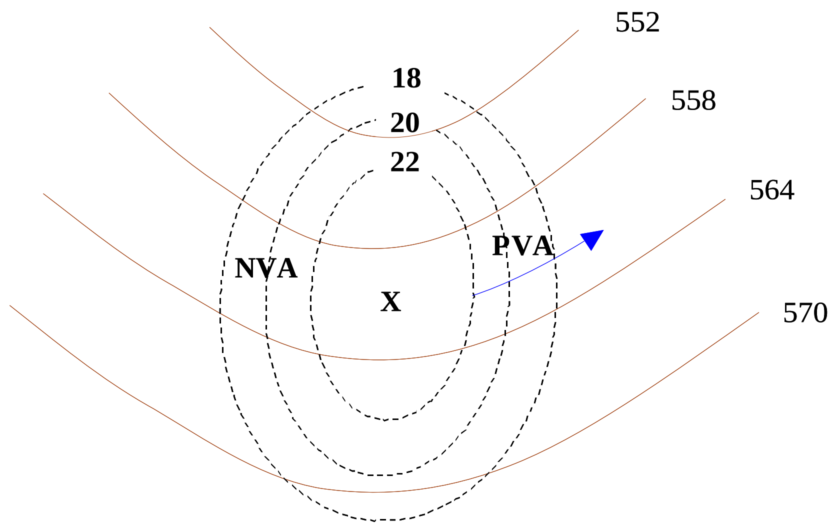
Divergence and convergence

In our numerous discussions of current and prognostic weather patterns, we have spoken of divergence and convergence in the horizontal fields of motion at different levels within the troposphere. Analysis of wave-like disturbances (Rossby waves) shows that the rate of change of vorticity relates to fields of divergence and convergence, as shown in the following diagram:



The curve labeled V represents the profile of wind speed through the troposphere, reaching a maximum at the tropopause (the jet stream). When $V > C + \beta L^2/4\pi^2$ as in the upper portion of the troposphere, convergence occurs in the horizontal wind field upwind of the trough line (west of the trough) and divergence occurs down wind of the trough. In the lower part of the troposphere ($V < C + \beta L^2/4\pi^2$), the opposite is the case. Compensatory vertical motions occur with ascent ahead of the trough (downwind) and subsidence behind the trough, together with the creation of areas of low and high pressure at the surface, as shown.

On weather maps which show upper air flow patterns, a way to identify divergence and convergence is to examine the relationship between height contours (say of 500 mb) and calculated vorticity values (such as plotted on the NGM forecast 500 mb maps)



As the air flows through the trough and moves downstream, it loses vorticity and is forced to diverge horizontally (recall that this is the same relationship as dictated by the conservation of angular momentum). Where the “boxes” formed by the crossing of the contours and the vorticity isopleths are smallest, divergence (or convergence on the upwind side of the trough) is greatest. Alternatively, as air moves downstream ahead of the trough, we can say that positive values of vorticity are advected by the air parcels carrying the positive vorticity acquired at the trough line. The downstream area is said to be one of positive vorticity advection (PVA), while the upstream region is one of negative vorticity advection (NVA). PVA is associated with upper air divergence, and NVA is associated with upper air convergence.