

Radiation laws

Blackbody radiation

Any substance (solid, liquid or gas) emits radiation according to its absolute temperature, measured in units of Kelvin ($K = ^\circ\text{C} + 273.15$). The efficiency at which radiation is emitted varies with the substance. However, at any temperature, there is an upper limit to how much radiation can be emitted.

A blackbody is a hypothetical body that:

- 1) Emits radiation at the maximum intensity possible for every wavelength.
- 2) Completely absorbs all incident radiation (hence the term "black").

Planck's Law

Describes the amount of radiation emitted by a blackbody at each wavelength as a function of temperature:

$$\text{Monochromatic irradiance } E_{\lambda}^* = \frac{C_1}{\lambda^5 \left[\exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]},$$

where we use the asterisk * to refer to blackbody radiation.

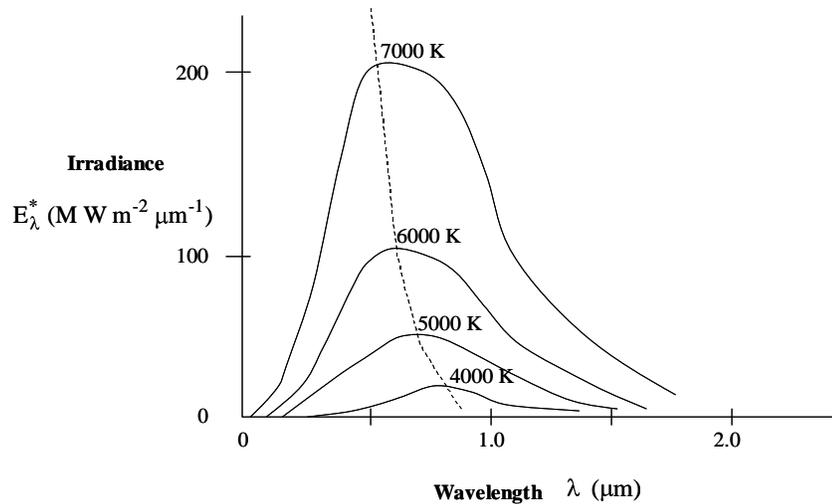
λ is the wavelength in meters

T is the absolute temperature in Kelvin (K)

$$C_1 = 3.74 \cdot 10^{-16} \text{ W m}^2$$

$$C_2 = 1.44 \cdot 10^{-2} \text{ m k}$$

Planck's law shows the spectral dependence of energy emitted by a blackbody at different absolute temperatures.



Note: The sun has an effective radiative temperature of about 6000 K but terrestrial objects have temperatures only of the order of 300 K.

At different radiative temperatures, the area under the Planck curve changes and the spectral quality of the radiation shifts along the wavelength axis.

These two effects are described by two well-known laws: one based on integration of the Planck equation to yield the total radiant energy emitted, and the other based on differentiation of the equation to obtain the peak wavelength of the spectrum.

(How does one obtain the total radiation emitted and how does one obtain the peak of the wavelength of the spectrum?)

Stefan-Boltzmann Law

The total amount of radiant energy emitted by a blackbody is proportional to the fourth power of the absolute temperature such that,

$$E^* = \sigma \cdot T^4 \quad \text{W m}^{-2}$$

where $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ is the Stefan-Boltzmann constant. E^* is the area under the E_λ^* curve, i.e.,

$$E^* = \int_0^\infty E_\lambda^* d\lambda$$

Wien's displacement Law

The wavelength of the peak in the spectral curve is given by

$$\lambda_{\max} = \frac{2897}{T} \quad \mu\text{m},$$

where T is the temperature in Kelvin.

The Wien displacement law shows that there is a wide separation between solar radiation (shortwave) and terrestrial radiation (longwave):

Solar ($T \approx 6000 \text{ K}$)	$\lambda_{\max} = \frac{2897}{6000} \approx 0.5 \mu\text{m}$
Terrestrial ($T \approx 300 \text{ K}$)	$\lambda_{\max} = \frac{2897}{300} \approx 10 \mu\text{m}$

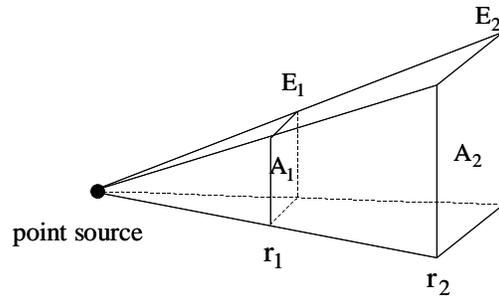
It is instructive to plot the spectra of solar and terrestrial radiation on the same diagram to show their relative contributions at different wavelengths. Before doing so, however, it is necessary to make the following changes:

- 1) Solar radiation is reduced according to the inverse square law to the mean orbital distance of earth.
 - 2) Solar radiation is further reduced (by a factor of 4) to express it in terms of unit surface area of the globe (surface area of a sphere is 4 times that of a circular disc).
 - 3) Wavelength is plotted on a log scale because of the large range of wavelengths involved in the two radiation streams.
 - 4) Irradiance is multiplied by the wavelength (λE_λ^*) so that equal areas on the graph respect equal amounts of radiant energy.
- Show the spectra of longwave and shortwave radiation (Fig. 6)

The solar “constant” S

The solar constant is the solar irradiance incident on a surface perpendicular to the sun’s ray at the mean orbital distance of the earth and outside the atmosphere. This represents the amount of solar radiation available for warming the earth and for driving its atmospheric and oceanic motions.

We can calculate the solar constant knowing its radiant output, its radius and the orbital distance of the earth, using the inverse square law.

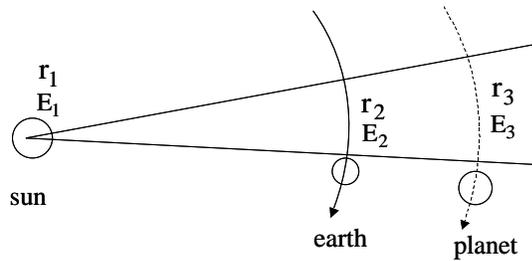


$$\frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2$$

If the same amount of radiant energy passes through areas A_1 and A_2 , the irradiance (flux of energy per unit area) is inversely proportional to the area, and thus to the squares of the radial distance from the origin.

$$\frac{E_2}{E_1} = \frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$

Now consider that r_1 is the radius of the sun and A_1 is an area on the surface of the sun which radiates with an effective temperature of 5784 K



Where $r_1 = 6.95 \times 10^5$ km, $r_2 = 149.6 \times 10^6$ km

From the Stefan-Boltzmann law,

$$E_1 = \sigma \cdot T^4 = (5.67 \times 10^{-8}) (5784)^4 \text{ W m}^{-2}$$

Then, using the inverse square law,

$$E_2 = E_1 \left(\frac{r_1}{r_2} \right)^2 = (5.67 \times 10^{-8}) (5784)^4 \left(\frac{6.95 \times 10^5}{149.6 \times 10^6} \right)^2$$

$$S = E_2 = 1370 \text{ W m}^{-2}$$

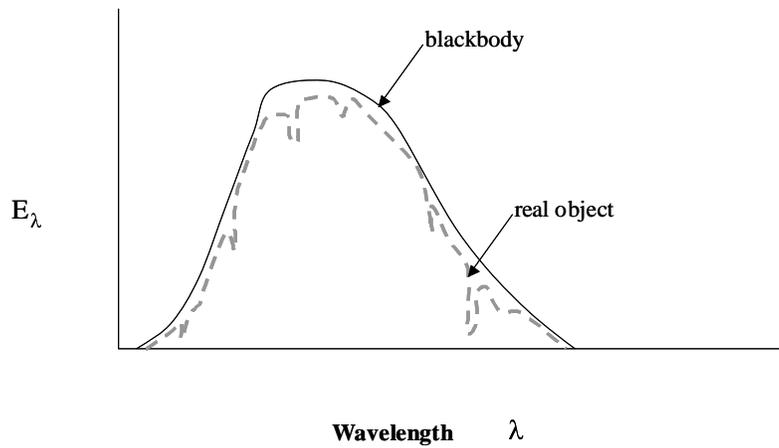
This is the solar constant, the irradiance of the sun at the earth's orbital distance.

Likewise, we could find the solar constant for planet x knowing its orbital distance, or we could calculate it based on knowledge of earth's solar constant.

$$\frac{E_3}{E_2} = \left(\frac{r_2}{r_3} \right)^2$$

Emissivity

Real objects emit radiation at a rate lower than that of a blackbody (a perfect emitter). The efficiency of emission by an object at a given temperature is wavelength dependent. For example,



Emissivity is defined as $\varepsilon_\lambda = \frac{E_\lambda}{E_\lambda^*} (\leq 1)$

But it is often convenient to express emissivity in integrated form, such as

$$\varepsilon = \frac{E}{E^*} = \frac{\int_0^\infty E_\lambda d\lambda}{\int_0^\infty E_\lambda^* d\lambda} = \frac{\int_0^\infty E_\lambda d\lambda}{\sigma T^4}$$

See the attached table of emissivities for various natural and artificial surfaces. (No table)

- Show an emissivity table?

Kirchhoff's law

Materials that are strong emitters at a particular wavelength are also strong absorbers at that wavelength such that

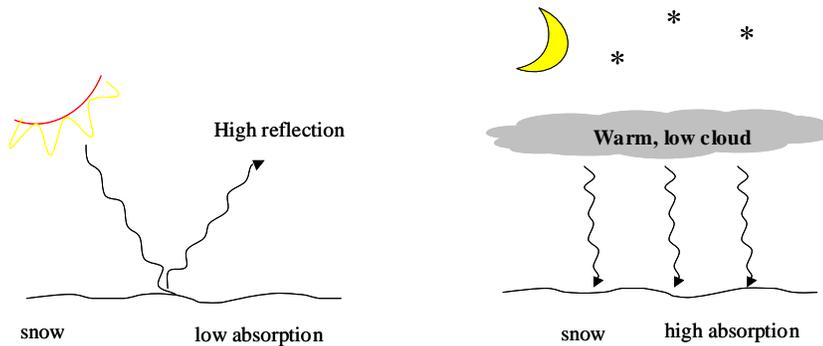
$$\epsilon_{\lambda} = a_{\lambda} ,$$

where a_{λ} is the absorptivity which is the fraction of incident radiation, at a given wavelength, that is absorbed by the surface. Note the wavelength dependence. We can only write the above expression in integration form

$$\epsilon = a$$

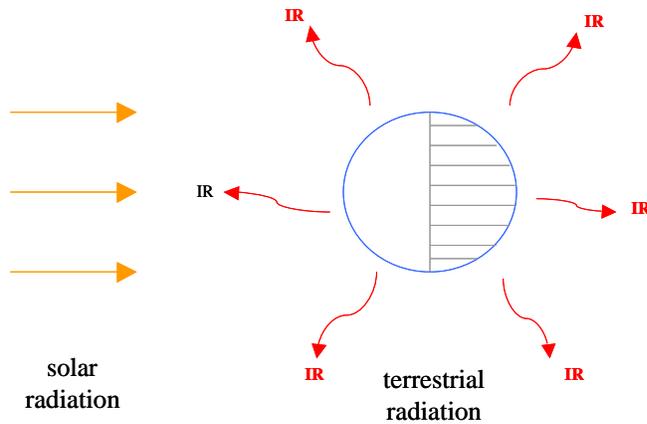
when the radiation emitted and the radiation absorbed have the same spectral distribution. One cannot equate the efficiency with which a surface absorbs solar radiation to the efficiency with which it emits terrestrial radiation.

For example, fresh snow has a high albedo (reflectivity to solar radiation) meaning a low absorptivity to solar radiation. Yet, snow is a very good emitter of longwave radiation (high emissivity). Under the right circumstances, snow can melt more quickly on a cloudy night than on a sunny day.



Energy balance of the earth/atmosphere system

Since radiant energy is the only mechanism by which the earth/atmosphere system can exchange energy with space, there must be a state of radiation balance (the earth loses as much radiant energy as it gains from the sun), assuming that the earth's temperature is approximately steady-state, and is not increasing or decreasing forever. This enables us to calculate an effective radiating temperature for the earth, as it would be observed from outer space.



The earth intercepts as a disc of radius r . The amount of solar radiation absorbed

$$= (1-\alpha)S\pi r^2,$$

where $\alpha = 0.3$ (albedo of earth/atmosphere)

$S = 1370 \text{ W m}^{-2}$ (solar constant)

The earth emits as a sphere at a rate given by the Stefan-Boltzmann law:

$$= \sigma \cdot T^4(4\pi r^2),$$

where T = equivalent blackbody temperature of the earth/atmosphere

σ = Stefan-Boltzmann constant

$$= 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

Equating the amount of solar radiation absorbed and the amount emitted, we obtain

$$(1-\alpha) \cdot S\pi r^2 = \sigma T^4 (4\pi r^2)$$

Giving

$$\begin{aligned} T &= \left[\frac{(1-\alpha) \cdot S}{4\sigma} \right]^{1/4} \\ &= \left[\frac{(1-0.3) \times 1370}{4 \times 5.67 \times 10^{-8}} \right]^{1/4} \\ &= 255 \text{ K } (-18 \text{ }^\circ\text{C}) \end{aligned}$$

This is a globally averaged effective radiating temperature and is probably equivalent to some upper atmospheric temperature. The earth's surface has an observed globally averaged temperature much warmer than this, close to 15 °C. The explanation for this is the well-know "greenhouse" effect.

Note: If we have chosen an emissivity for the earth such as $\epsilon=0.9$ to adjust the Stefan-Boltzmann radiation, we would have calculated an effective temperature of 262 K (-11 °C) still much colder than observed temperatures at the earth's surface.

Solar radiation

The attached diagram shows the spectral composition of solar radiation. Based on the position of the peak of the spectrum, the sun appears as a blackbody with temperature near 6000 K but absorption by gasses in the solar "atmosphere" reduces the overall emission, and the sun appears as an object with an effective or radiative temperature of 5784 K.

Solar irradiance changes with time, season, and location for the following reasons:

- 1) The solar "constant" changes due to changes in the output of the sun ($\pm 0.1\%$)
- 2) At different times of the year the distance between the sun and the earth is different because the earth's orbit is slightly elliptical. The sun and earth are closest in January (S-hemisphere summer) and furthest apart in July (N-hemisphere summer). The 1.7% difference in the distance translates to a 3.4% difference in irradiance because of the inverse square relationship.
- 3) Different parts of the earth are exposed to different amounts of solar radiation, depending on the time of the day and time of year, change of the zenith angle of the sun (angular distance from the overhead position).
- 4) The path length of solar radiation through the atmosphere varies as a function of zenith angle, causing depletion by scattering and absorption.

Radiation and the atmosphere

Of great importance is the difference between how the atmosphere transmits, absorbs, and scatters solar and terrestrial radiation streams. The most important statement that one could make is that the atmosphere is relatively transparent to solar radiation but is relatively opaque to terrestrial radiation.

In the absence of dense clouds, a large fraction of the incident solar radiation penetrates the atmosphere and reaches the surface. Most of the radiation (longwave) emitted by the earth surface is absorbed by the atmosphere but, because the atmosphere is a good absorber, it is also a good emitter (Kirchhoff's law) and reemits a large portion of the absorbed energy back to the surface. This is the "greenhouse effect".

Scattering of solar radiation

The scattering of sunlight in atmosphere is responsible for the blue color of the sky in clear conditions and also for the lack of color when the air is laden with pollutants (unless pollutants are also selective absorbers in which case the sky, especially the lowest layers when viewed from the side, can appear a murky brown).

The blue or colorless appearances of the sky result from two distinct types of scattering:

- 1) Rayleigh scattering is by particles (molecules and groups of molecules, in this case) much smaller than the wavelength of the radiation. Rayleigh scattering is very wavelength dependent such that the scattering coefficient (a measure of the efficiency of the scattering)

$$k \propto \frac{1}{\lambda^4}$$

For example, taking the wavelength of blue light to be $0.47\mu\text{m}$, and that of red light to be $0.64\mu\text{m}$,

$$\frac{k(\text{blue})}{k(\text{red})} \propto \left(\frac{0.64}{0.47}\right)^4 \cong 3.5$$

Thus, blue light is much more strongly scattered than red light, accounting for the blueness of the sky and for the red color of sunsets. The blue sky

color humans perceive is also related to the spectrum of radiation from the sun, which has less energy in the shorter violet wavelengths than in the blue wavelengths, and the sensitivity of the retina to colors.

- 2) Mie scattering is by particles of the same order of size as the wavelength (such as solid particles or small liquid droplets in polluted or hazy air), and is relatively independent of wavelength. As a result, the sky appears white or colorless when heavy with pollutants.

See also the following diagram presenting a general picture of scattering in the atmosphere.

- Show a scattering diagram

Transfer and absorption of solar radiation

Generally, a cloud-free atmosphere absorbs most solar radiation (short-wave) weakly. However, the shorter wavelengths ($\lambda < \approx 0.3\mu\text{m}$) are eliminated at high altitudes by N_2 , O_2 and O_3 . Strong absorption by ozone at about 50 km produces the temperature maximum at this height (the stratopause). In the near infrared, water vapor has several rather weak absorption bands. See the following two diagrams for details.

- Show a diagram for absorptivity

Clouds absorb some solar radiation but are mostly responsible for scattering and for reflection of solar energy back to space. The earth's albedo of 0.3 is largely a consequence of the amount of cloudiness.

Atmospheric absorption of terrestrial radiation

The spectral absorptivities shown in the following diagram clearly show the large differences between absorption and transmission of solar and terrestrial radiation. Again, the atmosphere is relatively transparent to solar radiation but nearly opaque to terrestrial radiation. Recalling that the spectrum of terrestrial radiation spans a range from about 4 μm to 100 μm with a peak at about 10 μm , we see that most radiation emanating from the earth surface would be absorbed by the atmosphere, with water vapor and CO_2 being the principle absorbers.

An atmosphere "window" appears in the absorption spectrum near the peak in the spectrum of terrestrial radiation ($\sim 10 \mu\text{m}$) allowing some longwave radiation to escape to space under cloud-free conditions. This "window" in this part of the spectrum is utilized by satellite IR imagery to observe cloud cover and other features both day and night.

Clouds are very strong absorbers of longwave radiation.

The earth/atmosphere energy budget

Being a good absorber of longwave radiation, and according to Kirchhoff's law, the atmosphere is also an excellent emitter of radiation of the same spectral composition. Clouds are also good emitters of terrestrial radiation according to their temperature, so warm (usually low) clouds emit more strongly than cold (usually high) clouds, as dictated by the Stefan-Boltzmann law.

This absorption and emission of terrestrial radiation plays a large role in governing the energy balance of the earth/atmosphere system and is the reason that changes in the concentration of "greenhouse" gasses are likely to have an impact on global climate.

The following diagram summarizes, in a globally averaged manner, the exchanges of energy that take place between the earth's surface, the atmosphere, and space. All the numbers are normalized to 100 units of incoming solar radiation. Note the large magnitudes associated with longwave radiation, some of the numbers even larger than that of the ultimate energy source, the sun.

Radiative Energy Budget

Earth/atmospheric system

Net solar absorbed	70	
Net longwave emitted	-70	
	<hr/>	
	0	

Earth surface

Net solar absorbed	45	
Net longwave emitted	-16	
	<hr/>	
	29	net gain

Atmosphere

Net solar absorbed	25	
Net longwave emitted	-54	
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	-29	net loss

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