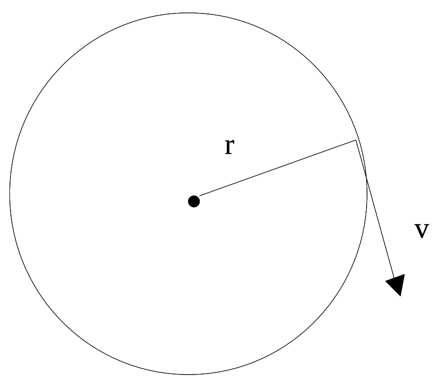
Rotation

The atmosphere rotates with the earth and motions within the atmosphere clearly follow curved paths (cyclones, anticyclones, hurricanes, tornadoes etc.)

We need to express rotation quantitatively. For a solid object or any mass that does not distort during rotation, we use the term angular velocity ω (Greek omega). Later, we will assign the term vorticity to the rotation of a fluid such as the atmosphere.

Angular velocity ω expresses the rate of rotation of a body and is defined as the angle in radians through which the body turns in unit time. The units of ω are therefore radians/second. Remember though that a radian is a fraction of a complete revolution (2π radians $\equiv$ 360^o) and it is not a fundamental dimension (like time or length) that must be carried through in any calculation involving ω.

An object moving with speed v around a point at distance r, has an angular velocity

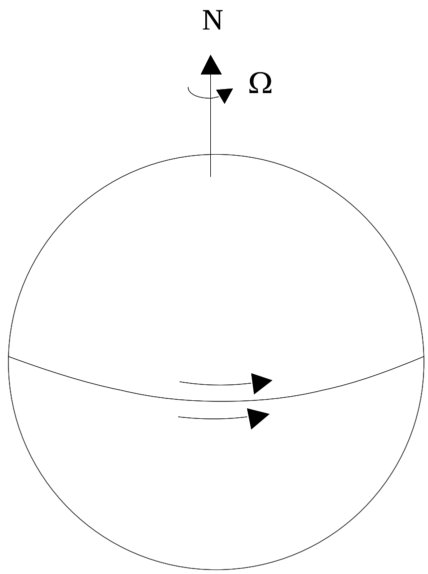


$$\omega = \frac{v}{r} \quad \left[\frac{m/s}{m} \equiv s^{-1}\right]$$

Angular velocity is a vector quantity. We adopt the convention that the vector associated with the rotation is aligned with the axis of rotation in the sense of a right-hand screw (the right hand rule: with your thumb aligned with axis of rotation and pointing in the direction of the vector, your fingers, curling around the palm of your hand, will give the direction of rotation).

The angular velocity of the earth Ω

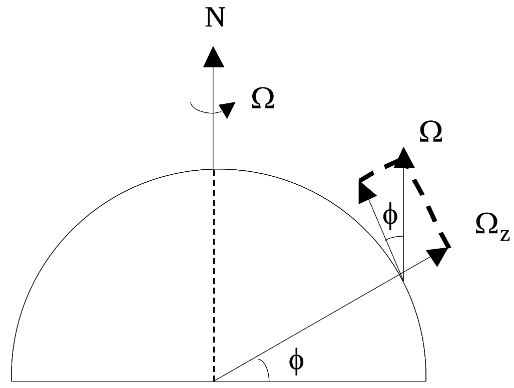
The earth rotates towards the east and, therefore, the vector angular velocity points outwards from the north pole (and into the south pole). The magnitude of Ω is one revolution (2π radians) per day but we must calculate it from the duration of the sidereal day, relative to the stars and not to the sun. The sidereal day is 23 hrs 56 minutes



$$\Omega = \frac{2\pi}{(23\,\text{hrs}\;56\,\text{min})}$$

$$= 7.292\ 10^{-5}\ \text{rad/s}$$

Since most atmosphere motion is parallel to the earth’s surface and the atmosphere is a very thin sheet encircling the globe, we are concerned primarily with the magnitude of the component of the earth’s angular velocity along the local vertical direction. This is rotation in a local horizontal plane.



φ is latitude angle

Ω_z is the local vertical component of the angular velocity.

At the north pole, $\Omega_z = \Omega$ (-Ω at the S-pole)
At the equator, $\Omega_z = 0$

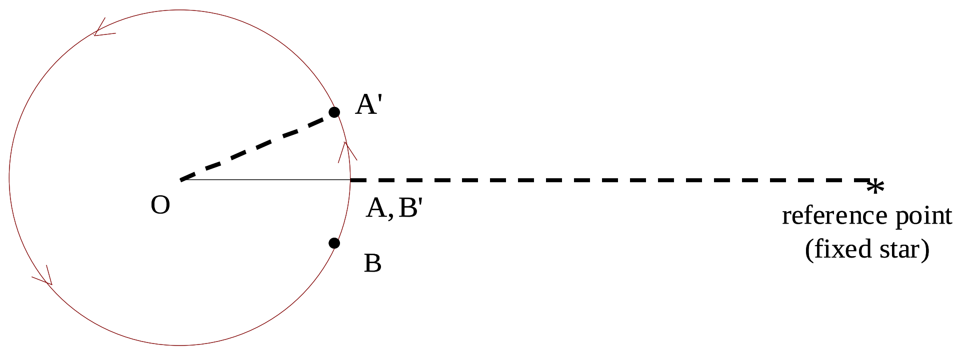
In general, $\Omega_z = \Omega \sin\phi$.

The Coriolis effect

The Coriolis effect is the apparent deviation of any body or mass of air in motion that results from the rotation of the earth. The atmosphere is coupled to the earth only through gravity and friction at the surface. Otherwise, it is free to move independently.

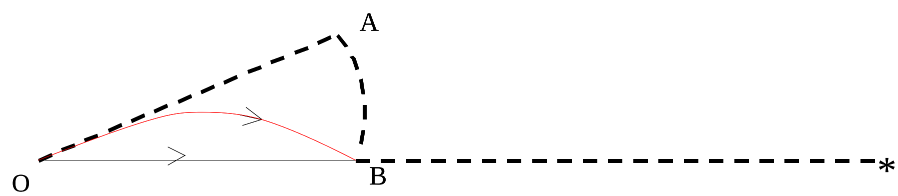
Any analysis of the motion of the atmosphere in terms of Newton's laws of motion must be made in a frame of reference oriented to the stars (in inertial frame of reference). However, the earth rotates in this frame of reference and is therefore a non-inertial frame. To compensate for the influence of the earth's rotation, we introduce an apparent force, the Coriolis force, that allows us to treat the atmosphere using the basic laws of motion.

Consider an object (or a mass of air) moving on a surface that is rotating about some local vertical axis with angular velocity ω. Suppose the object is not subject to any external forces and is thus moving at constant speed towards some reference point outside the rotating surface.



At the start, the point A is aligned with the reference point and the object heads directly for it. However, in the interval of time it takes for the object to reach this radial distance, point A has rotated to A’. The object arrives at point B on the surface which has rotated to B’, in line with the reference point.

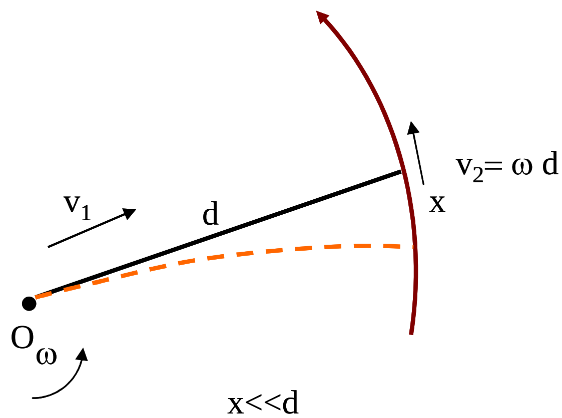
Analyzing the motion as an observer on the rotating surface, we would see the object moving in a curved path as shown on the plot below.



The curved path O to B is as seen by the surface bound observer, while the straight line O to B is the path as viewed by an observer outside the rotation coordinate system. The curved path appears as a deviation to the right of the motion in this case in which the rotation is anticlockwise (as is the rotation of the northern hemisphere). In the rotating frame of reference, we can “explain” this motion by introducing the Coriolis force and Coriolis acceleration.

The magnitude of the Coriolis acceleration

Consider an object in motion in the northern hemisphere subject only to the Coriolis force acting to turn it to the right from its current direction of motion.



Let Coriolis acceleration be a, then the distance traveled to the right

$$x = \frac{1}{2} a \cdot t^2$$

where t is the travel time, which can be related to the speed of the object v and the distance traveled d such that

$$t = \frac{d}{v_1}$$

and, therefore

$$x = \frac{1}{2} a \cdot \left(\frac{d}{v_1} \right)^2$$

Now, the sideways motion can also be calculated from the angular velocity of the target about the origin. Let this angular velocity be ω, then

$$x = v_2 t = \omega \cdot d \cdot t = \frac{\omega d^2}{v_1}$$

Equating these two expressions for x, we obtain

$$x = \frac{1}{2} a \cdot \left(\frac{d}{v_1} \right)^2 = \frac{\omega d^2}{v_1}$$

giving

$$a = 2\omega v_1 \qquad \text{(no motion, no force)}$$

If ω is that due to the earth's rotation about a local vertical axis, such that

$$\omega = \Omega \sin\phi$$

then

$$a = (2\Omega \sin\phi) v_1$$
$$= f v_1$$

where $f = 2\Omega \sin\phi$ is called the Coriolis parameter

Calculating f at different latitudes,

Latitude	$f(s^{-1})$
0	0
15	$3.8\ 10^{-5}$
30	$7.3\ 10^{-5}$
45	$10.3\ 10^{-5}$ (mid latitudes $\approx 10^{-4}$)
60	$12.6\ 10^{-5}$
75	$14.1\ 10^{-5}$
90	$14.6\ 10^{-5}$ (twice Ω)