The conservation of angular momentum

The quantity $mvr\ (= m\omega r^2)$ is called angular momentum. It is a consequence of the law of conservation of energy that, in the absence of external forces (torques), angular momentum is conserved, such that

$$mvr = m\omega r^2 = \text{constant}$$

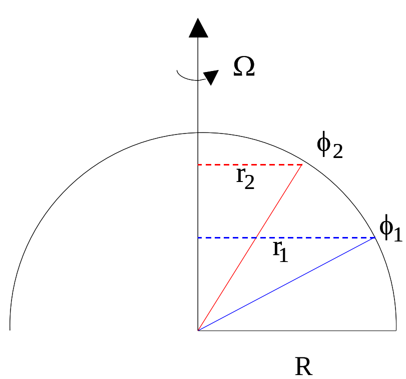
If an object is held to move in a circular path (an applied force maintains a centripetal acceleration), and if the radius of rotation decreases to one half its original value, what happens to the linear and angular velocities, v and ω, respectively?

Examining the expression above, we see that the velocity v will double and the angular velocity ω will increase by a factor of four.

There are many examples from everyday experience:

- the spinning figure skater
- an object on a string, whirled around and brought closer to the center of rotation
- water draining from a basin

There are also important examples in meteorology. The conservation of angular momentum explains the Coriolis effect. Consider air moving northward in the northern hemisphere. Initially it is rotating about the axis of rotation of the earth with the same angular velocity as that of the earth itself, $\omega = \Omega$. As it northward, its radius of rotation about the axis of the earth is reduced. Hence, according to the law of conservation of angular momentum, its angular velocity must increase.



Let the air move due north from latitude ϕ_1. Initially, its radius of rotation is

$$r_1 = R\cos\phi_1 .$$

Its angular momentum (per unit mass) is

$$\omega_1\ r_1{}^2 = \Omega\ r_1{}^2 = \Omega\ (R\cos\phi_1)^2$$

At latitude ϕ_2,

$$\omega_2\ r_2{}^2 = \omega_2\ (R\cos\phi_2)^2$$

which must equal the original value, thus

$$\omega_2\ (R\cos\phi_2)^2 = \Omega\ (R\cos\phi_1)^2$$

$$\omega_2 = \Omega\ \left(\frac{\cos\phi_1}{\cos\phi_2}\right)^2$$

and $\omega_2 > \Omega$

and the air rotates around the axis of the earth move quickly than does the earth itself. An observer would see the air as having a component to the east, which is a deviation to the right from its original path.

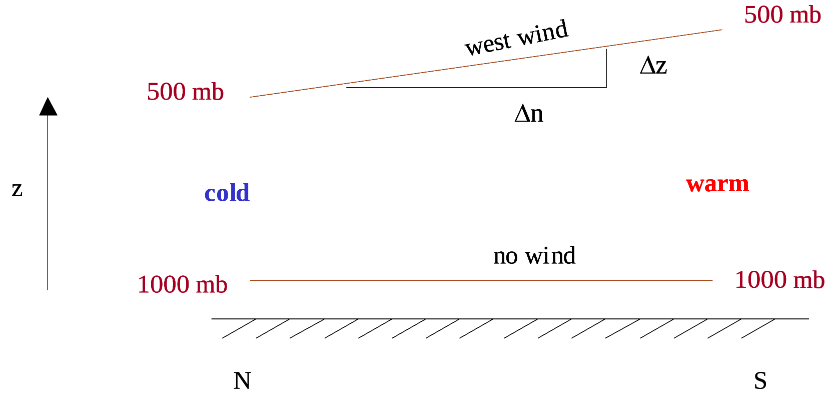
It is straightforward to show that the law will also explain the departure to the right (towards the west) of an object moving towards the south.

Horizontal divergence and convergence are common features of atmospheric flow. If air that is rotating is forced to converge, there will be a concentration of such rotation and consequent “spin up”. Examples are dust devils, tornadoes, water spouts, and cyclones of all scales.

The thermal wind

We have seen many examples of weather charts showing the air flow patterns at different levels in the atmosphere. The link between patterns at different levels has been stressed but now we can formalize that link by evaluating the difference in height of two pressure surfaces. We do this by introducing the concept of the thermal wind.

The thermal wind is not a wind that can be measured with an instrument but rather is the (vector) difference between the wind at two levels (pressure levels). Consider, first, the simple case of no wind near the surface (the 1000 mb surface is horizontal) and a temperature pattern with a uniform north-to-south gradient up to the 500 mb level.

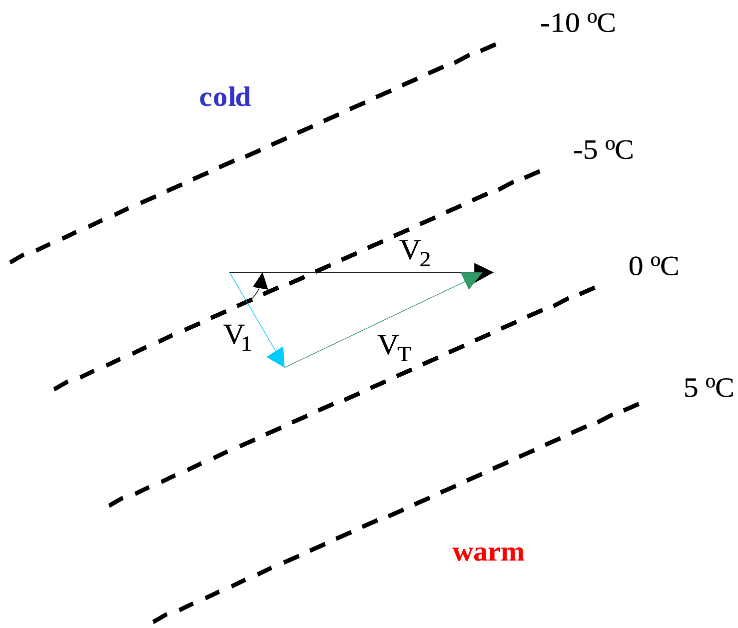


The vertical separation between the two pressure levels is called the thickness (in this case, the 1000 to 500 mb thickness). Now, pressure is a consequence of the mass of atmosphere above the level in question. Thus, between 1000 and 500 mb, there is a fixed mass of atmosphere. The thickness of this mass is a function of the density of the air which, in turn, is a function only of the mean temperature of the layer. In regions of warm air (as in the south end of the diagram) the thickness is large, while, where temperatures are low, the thickness is small.

The slope to the 500 mb surface seen in the diagram is therefore a direct consequence of the distribution of temperature between these two pressure levels. Knowing the 1000 mb height field, and knowing this distribution of temperature, we can determine the 500 mb height field.

Now, at 500 mb the diagram shows a slope upwards towards, the south. Our knowledge of the geostrophic wind tells us that the wind will be from the west (into the paper with low heights on the left). The difference between the wind at the surface and the wind at the 500 mb level is a consequence of the difference in the slopes of the 1000 and 500 mb surfaces, which, in turn is a consequence of the temperature gradient. The difference in wind between the two levels (the thermal wind) relates to the temperature pattern (low temperature on the left) the same way that the geostrophic wind relates to the pressure pattern (low pressure on the left).

This rule applies in general, when the low level wind is not zero and when the wind is not aligned with the isotherms. The diagram below shows this:



Here, V_1 is the wind at the lower level. The temperature pattern shows isotherms of mean temperature between the two levels. V_T is the thermal wind “blowing” parallel to the isotherms (and proportional to the temperature gradient).

The vector addition of V_1 and V_T yields the wind at the upper level, V_2. Wind patterns at two levels are directly linked by the distribution of temperature!

As an example of the use of this relationship, we can derive a simple forecasting rule. Notice that, in the example in the diagram above, the wind at both levels (V_1 and V_2) both blow from cold to warm and temperatures at any fixed location would be expected to decrease with time. The wind changes from V_1 to V_2 by turning in an anticlockwise sense. This is called backing. Backing is associated with a cooling trend as we see above, while veering (wind turning clockwise with height) is associated with a warming trend. The student should verify this.

Sometimes, one can observe a shift in the wind between two levels, perhaps by the motions of clouds at different height and it is possible to draw a conclusion regarding the likelihood of a warming or cooling trend over the next 24 hours or so.

Mid-latitude westerlies and the jet stream

The thermal wind relationship explains to us why the wind is predominantly westerly in mid latitudes and why the highest westerly winds are found in the upper troposphere.

As in the simple diagram in the figure before the previous one, a westerly wind will build up as the pressure surfaces increase in slope with increasing height due to the relative warmth of the low latitudes and the relative coldness of the high latitudes. The following diagrams illustrate this. Highest winds tend to be concentrated in a jet (the jetstream) at the top of the troposphere. Above the tropopause, in the lower stratosphere, the temperature gradient reverses (lowest temperature over low latitudes), the thermal wind reverses in direction, subtracting off from the magnitude of the westerly jet, and winds diminish with height.

The diagrams show monthly or seasonal averages which reduce the magnitude of the jet. At any one time and location, the jet can be much stronger than those shown here. Notice the seasonal change in the position and strength of the jet.