Adiabatic processes

We can see a simple relationship between changes in pressure and temperature for cases when dQ=0, i.e., no heat is added or removed from the system. As it turns out, many atmospheric processes approximate this situation. Such a process called adiabatic – a system undergoes changes (in pressure, say) but no heat is allowed to enter or leave the system.

Examples of processes in the atmosphere that closely approximate being adiabatic are:

- Orographic lifting
- Large scale convection
- Large scale lifting or subsidence

In which mixing or energy exchange with the surroundings occurs relatively slowly compared with the speed of the process itself. Can you suggest diabatic processed in which $dQ \neq 0$?

Poisson's equation

For an adiabatic process, dQ=0 and the first law can be written

$$0 = C_p dT - \alpha dp$$

and, rearranging,

$$dT = \frac{\alpha dp}{C_p}$$

The variable α can be eliminated, using the equation of state

$$\alpha = \frac{RT}{p}$$

such that

$$dT = \frac{RT}{C_p}\frac{dp}{p}$$

or

$$\frac{dT}{T} = \frac{R}{C_p}\frac{dp}{p}$$

and, on integration

$$\ln T - \ln T_o = \frac{R}{C_p}(\ln p - \ln p_o)$$

or

$$T = T_o \left(\frac{p}{p_o} \right)^{R/c_p}$$

For dry air, R = 287 J/kg/K
and C_p = 1004 J/kg/K

So,

$$\frac{R}{C_p} = \frac{287}{1004} = 0.286$$

Giving $$T = T_o \left(\frac{p}{p_o} \right)^{0.286}$$

This is Poisson's equation defining how temperature changes for a change in pressure during an adiabatic process.

Potential temperature, θ

We define the "potential temperature" of a parcel of air as the temperature that would be achieved by bring the parcel dry adiabatically to a pressure of 1000 mb. If the initial temperature and pressure of an air parcel were T and p, its potential temperature θ is obtained from Possion's equation such that

$$\theta = T \left(\frac{1000}{p} \right)^{0.286}$$ p is in the unit of mb

The adiabatic lapse rate

A lapse rate is a rate of decrease of temperature with height (-dT/dz) either:

1) Observed by an instrument mounted on a balloon (a radiosonde) as it rises through the atmosphere, or
2) Experienced by a "parcel" of air as it physically or hypothetically rises towards levels of lower pressure.

The adiabatic lapse rate is the lapse rate of a dry parcel of air rising adiabatically through the atmosphere. More accurately we should call this the dry adiabatic lapse rate to distinguish it from a process in which condensation or evaporation of water droplets is occurring (the moist or saturated adiabatic lapse rate).

We can determine the magnitude of the dry adiabatic lapse rate by combing the hydrostatic equation and the first law of thermodynamics for an adiabatic system.

The hydrostatic equation ca be written in terms of either density or specific volume:

$$\frac{dp}{dz} = -\rho g = -\frac{g}{\alpha}$$

rearranging this: $\alpha dp = -g dz$

Now, one form of the first law of thermodynamics for an adiabatic process (dQ=0) is

$$0 = C_p dT - \alpha dp$$

So, if we substitute for αdp, we obtain

$$0 = C_p dT - g dz$$

which we can rearrange to become

$$\frac{dT}{dz} = -\frac{g}{C_p}$$

Now, $g = 9.81\ m\ s^{-2}$, and $C_p = 1004\ J\ kg^{-1}\ K^{-1}$

thus,

$$\frac{dT}{dz} = -\frac{9.81}{1004}\ {}^{o}C/m = -9.8^{o}C/km$$

If a “parcel” of air is lifted in the atmosphere such that it does not exchange energy nor mix with its surroundings, it will decrease in temperature with height at the adiabatic lapse rate as calculated.

$$\Gamma_d = -\frac{dT}{dz} = 9.8^{o}C/km$$

This is an important reference lapse rate for atmospheric processes.

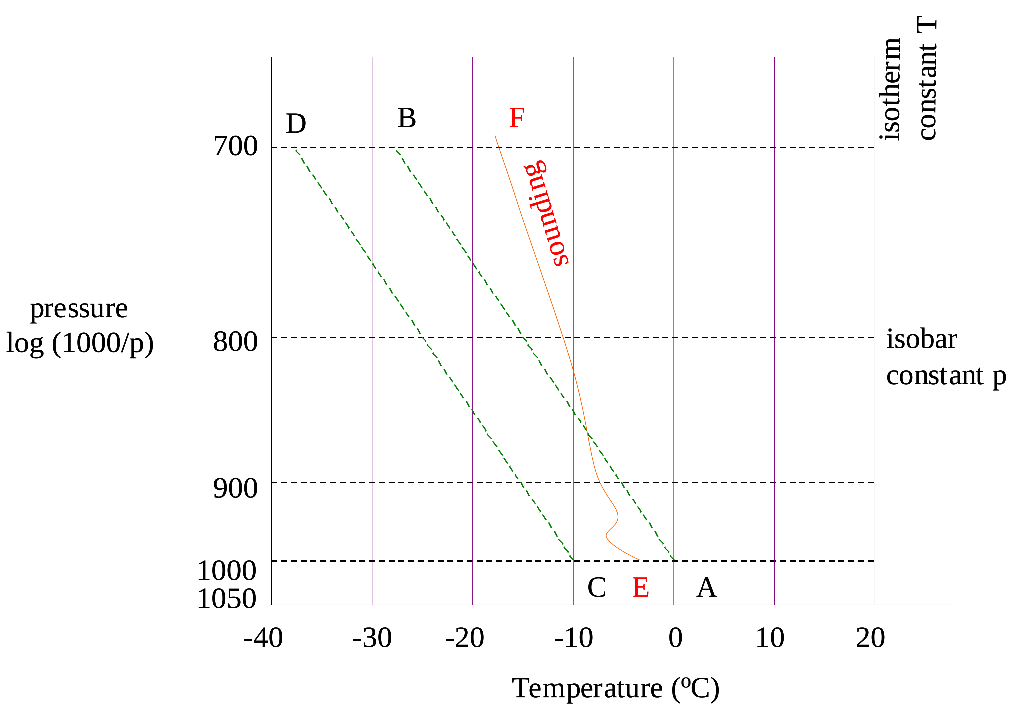
Thermodynamic diagrams

A diagram on which variations of the thermal state of a system are shown is called a thermodynamic diagram.

Since the equation of state relates the three state variables, a system is completely specified by any two of the variables (if you know any tow, you can calculate the third from the equation of state).

For example, in physics or engineering, a diagram to study heat engines plots pressure against volume. On a P-V plot, the work done in a cyclic process is represented by the area enclosed by the curve.

In atmospheric science, it is usual to construct diagrams using the two measured state variables (pressure and temperature). Since pressure changes (decreases) with height, it is plotted on the vertical axis with lowest values at the top and highest values at the bottom. Further, to retain the work-area property it is necessary to plot the pressure on a logarithmic scale. This is convenient because it makes the vertical axis a nearly linear function of height.



Process curves and sounding curves

Thermodynamic charts or diagrams are used to display both process and sounding curves. The distinction is important.

A process curve is a line drawn to represent a specific process such as adiabatic ascent or descent in the atmosphere. This could be a hypothetical process that we use for reference purpose, or it could be a process that represents a real event in the atmosphere. Many atmospheric processes are approximately adiabatic and meteorological thermodynamic diagram always include adiabats (in the case dry adiabats) for reference purposes. The diagram includes two dry adiabats (lines AB and CD). A real diagram will include many such lines.

Suppose a parcel with initial temperature 0 ^{o}C at the 1000 mb level is caused to rise dry adiabatically. As it rises to lower pressure it will cool at the rate specified by the line AB. Similarly, a parcel lifted from point C will cool at the rate given by the dry adiabat CD.

The adiabats are defined by Poisson's equation, relating pressure and temperature for a dry adiabatic process

$$T = T_o \left(\frac{p}{p_o} \right)^{0.286} \quad \text{T is in Kelvin}$$

It is convenient to label each of the dry adiabats on a diagram according to the temperature achieved as each line passes through the pressure level of 1000 mb (points A & C on the diagram).

Thus we define the "potential temperature" of a parcel of air as the temperature that would be achieved by bring the parcel dry adiabatically to a pressure of 1000 mb. If the initial temperature and pressure of an air parcel were T and p, its potential temperature θ is obtained from Possion's equation such that

$$\theta = T \left(\frac{1000}{p} \right)^{0.286} \quad \text{p is in the unit of mb}$$

Example:

The US standard atmosphere (Appendix H of text) lists the temperature and pressure at a height of 5 km to be 255.7 K (-17.5 ^{o}C and 540.5 mb, respectively. The potential temperature of the air at this height is computed from

$$\theta = 255.7\left(\frac{1000}{540.5}\right)^{0.286} = 304.9\text{K}$$ (31.7 ^{o}C or 89.1 ^{o}F)

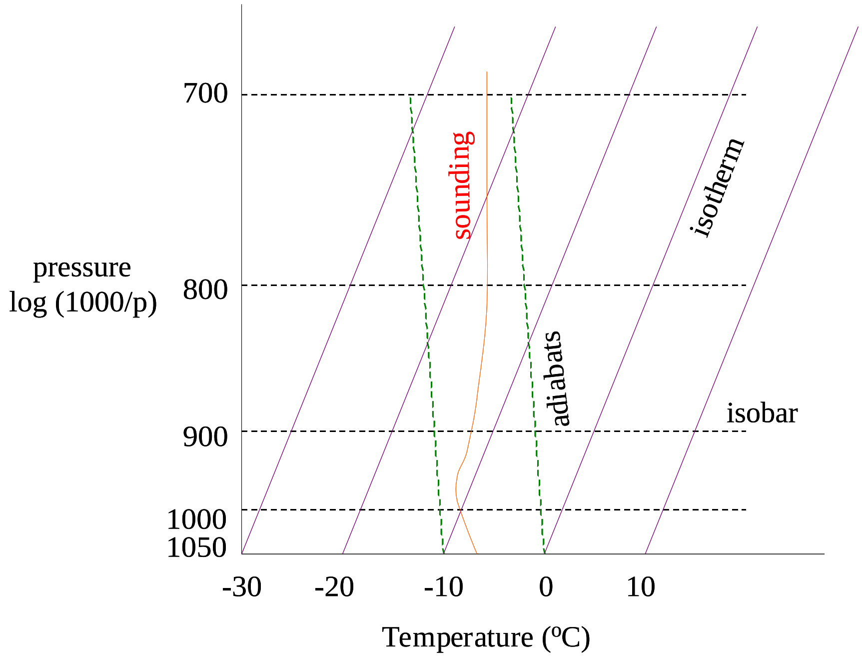
This is the temperature that would be achieved if a parcel of air from 5km were brought dry adiabatically down to 1000 mb.

Notice that, along any single dry adiabat, the potential temperature does not change.

Soundings are also plotted on thermodynamic diagrams. An example is the line EF on the diagram. A sounding is a plot of observational data collected usually with a radiosonde, a balloon borne package that measures temperature, pressure, and humidity and transmits this data back to the surface as it rises. Radar tracking of the balloon also provides information on wind speed and direction at heights within the atmosphere. A comparison of soundings and process curves reveals information about atmospheric stability.

Skew T-log p diagrams

Skew-T log-p diagrams are used extensively in meteorology and the class will have the opportunity to work with them during one fo our discussion groups. The skew T-log p is a modification of the earlier diagram which accommodates the fact that temperature decreases with height through the troposphere. To more economically use the diagram, the isotherms are “skewed” to the right with increasing height so that soundings and process curves are more vertical (making better use of the paper). Thus,


700
800
900
1000
1050
pressure
log (1000/p)
sounding
isotherm
adiabats
isobar
-30
-20
-10
0
10
Temperature (ºC)