Atmospheric Moisture

Measures of Humidity

a) Density of water vapor (absolute humidity) (ρ_v)

$$\rho_v = \frac{m_v}{V}$$

m_v : mass of water vapor

V : total volume of air

b) Vapor pressure (e)

The partial pressure exerted by the molecules of vapor in the air.

$$e = \rho_v R_v T$$

c) Saturation vapor pressure (e_s)

The vapor pressure when in equilibrium with a plane surface of pure water. Only function of T.

$$\ln \frac{e_s \text{ (Pa)}}{611 \text{ (Pa)}} = \frac{L}{R_v}\left(\frac{1}{273.15} - \frac{1}{T}\right),$$

where L is the latent heat of vaporization at 0^oC and its value is 2.5×10^6 J/kg. R_v, the gas constant of water vapor, is 461 J/K/kg.

e_s (water) > e_s (ice) ? or e_s (water) < e_s (ice) ?

d) Mixing ratio (ω)

The ratio of the mass of water vapor to the mass of the dry components of the atmosphere in a given volume. Usually expressed in g/kg

$$\omega = \frac{m_v}{m_d}$$

$$= \frac{m_v / V}{m_d / V} = \frac{\rho_v}{\rho_d} = \frac{e / R_v / T}{p_d / R_d / T}$$

$$= \frac{e / R_v / T}{(p - e) / R_d / T} = \frac{e}{(p - e)} \frac{R_d}{R_v}$$

$$= 0.622 \frac{e}{(p - e)}$$

m_d: mass of dry air

e) Saturation mixing ratio (ω_s)

$$w_s = 0.622 \frac{e_s}{(p - e_s)}$$

f) Specific humidity (q)

The ratio of the mass of water vapor to the mass of the total mass of the air in a given volume. Usually expressed in g/kg

$$q = \frac{m_v}{m_{air}} = \frac{m_v}{m_v + m_d} = 0.622 \frac{e}{p}$$

g) Relative humidity

$$RH = 100 \frac{e}{e_s} \% \quad \text{or} \quad RH = 100 \frac{\omega}{\omega_s} \%$$

h) Dew point (temperature) (T_d)

The temperature at which saturation would occur if moist air was cooled isobarically (at constant pressure)

☺ Given T and T_d, can you calculate e and e_s?

i) Wet Bulb Temperature (T_w)

The lowest temperature that can be reached by evaporating water into the air.

☺ What is the order of the magnitudes of T, T_d, and T_w?

Virtual temperature (T_v)

Rather than use a gas constant for moist air, it is more convenient to retain the gas constant for dry air and use a factious temperature (called virtual temperature) in the ideal gas equation.

The virtual temperature is the temperature that dry air must have in order to have the same density as the moist air at the same pressure.

$$
\begin{aligned}
p &= \rho R T \\
&= \rho R_d T_v
\end{aligned}
$$

$T_v > T$?   or $T > T_v$?

$$
\begin{aligned}
e &= \rho_v R_v T \\
p_d &= \rho_d R_d T \\
p &= p_d + e \\
\rho &= \rho_d + \rho_v \\
&= \frac{p-e}{R_d T} + \frac{e}{R_v T} = \frac{p}{R_d T} - \frac{e}{R_d T} + \frac{e}{R_v T} \\
&= \frac{p}{R_d T} - \frac{eP}{R_d TP} + \frac{eP}{R_v R_d TP / R_d} = \frac{p}{R_d T}\left(1 - \frac{e}{p} + \frac{e}{P}\frac{R_d}{R_v}\right) \\
&= \frac{p}{R_d T}\left[1 - \frac{e}{p}\left(1 - \frac{R_d}{R_v}\right)\right] \\
p &= \rho R_d \frac{T}{\left[1 - \frac{e}{p}\left(1 - \frac{R_d}{R_v}\right)\right]} \\
&= \rho R_d T_v
\end{aligned}
$$

$$
T_v = \frac{T}{\left[1 - \frac{e}{p}\left(1 - \frac{R_d}{R_v}\right)\right]}
$$

If T = 300 K, p = 1000 m, and e = 9 mb, what is the vale of T_v ?

Moist air is less dense than dry air; therefore, the T_v is always greater than T. However, even for very warm, moist air the T_v exceeds the T by only a few degrees.

If T_v is a constant, p is proportional to ρ.